4.2 Irrational Numbers

LESSON FOCUS Identify and order irrational numbers.

Make Connections

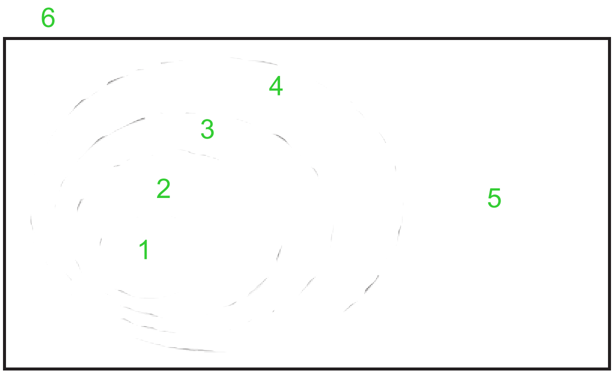
The formulas for the area and circumference of a circle involve π, which is not a rational number because it cannot be written as a quotient of integers.

What other numbers are not rational?

Friday, September 23rd

- Review Number Terminology
- Finish notes/examples
- Practice terminology (chart)
- Number Activity
- Classwork

Together, the rational numbers and irrational numbers form the set of **real numbers**. This diagram shows how these number systems are related.



4.2 Irrational Numbers

- Definitions

- Real numbers (R): ALL numbers; rational & irrational
- Irrational numbers ($\overline{Q}$):

  -they cannot be written as a fraction
  -non-repeating decimal
  -non-terminating decimal

  -Examples: 0.2163875943.... and π
- Rational numbers (Q):

  -a number that can be written as a fraction
  -Any number that is not an irrational number
  -Examples: -2.34, 3.$\overline{456}$, 6.323 232 32...

Definitions continued...

- Integers (I):
  - -Positive and negative whole numbers
  - -NO decimals
  - -Examples: -400, +8, 0, 29, -49578
- Whole numbers (W):
  - -all of the positive integers and zero
  - -Examples: 0, 1, 2, 3, 4, etc.
  - -NO decimals
- Natural numbers (N):
  - -all of the positive integers
  - -DOES NOT include zero (only difference from whole numbers)
  - -Examples: 1, 2, 3, 4, etc.

THE NUMBER SYSTEM

W =	Whole Numbers	I =	Integers
$\overline{Q}$ =	Irrational Numbers	R =	Real Numbers
N =	Natural Numbers	Q =	Rational Numbers

EXAMPLES:

W: 0, 1, 2, 3, ...

$\overline{Q}$: π (3.141592…), $\sqrt{3}$, 1.23456738… , $\sqrt{15}$, …

N: 1, 2, 3, ...

I: ....-3, -2, -1, 0, 1, 2, 3, …

R: $-\frac{1}{2}$, $\sqrt{15}$, 0 , -3 , 3 , π (3.141592) , …

Q: $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{11}{3}$, 0.2 , -0.2 , 3 , -3 , 0 , …

Copy and complete the table:

For each of the following numbers in the table, put an "x" in each category that the number belongs to. It may only belong in one, but could also belong to 5 out of the 6 categories. The first one is done for you.

Number	Real	Rational	Irrational	Whole	Natural	Integer
3.2	x	x				
0						
5.66						
-7						
15						
20009						
4.569...						
3.14...						
-3.22						
4/5						
14/2						
-6/3						
5/2						
-4.567...						
-23						
10						

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TRY THIS

Work with a partner.

These are rational numbers.	These are not rational numbers.
$\sqrt{100}$ $\sqrt{0.25}$ $\sqrt[3]{8}$ 0.5	$\sqrt{0.24}$ $\sqrt[3]{9}$ $\sqrt{2}$
$\frac{5}{6}$ $\sqrt{\frac{9}{64}}$ 0.8^2 $\sqrt[5]{-32}$	$\sqrt{\frac{1}{3}}$ $\sqrt[4]{12}$

A. How are radicals that are rational numbers different from radicals that are not rational numbers?

B. Which of these radicals are rational numbers?
Which are not rational numbers? How do you know?

$\sqrt{1.44}$, $\sqrt{\frac{64}{81}}$, $\sqrt[3]{-27}$, $\sqrt{\frac{4}{5}}$, $\sqrt{5}$

C. Write 3 other radicals that are rational numbers.
Why are they rational?

D. Write 3 other radicals that are not rational numbers.
Why are they not rational?

How are radicals that are rational numbers different from radicals that are not rational numbers?

Which of these radicals are rational numbers?
Which are not rational numbers? How do you know?

$\sqrt{1.44}$ $\sqrt{\frac{64}{81}}$ $\sqrt[3]{-27}$ $\sqrt{\frac{4}{5}}$ $\sqrt{5}$

Write 3 other radicals that are rational numbers. Why are they rational?

Write 3 other radicals that are not rational numbers. Why are they not rational?

Radicals that are square roots of perfect squares, cube roots of perfect cubes, and so on are rational numbers. Rational numbers have decimal representations that either terminate or repeat.

?

When an irrational number is written as a radical, the radical is the *exact value* of the irrational number; for example, $\sqrt{2}$ and $\sqrt[3]{-50}$. We can use the square root and cube root keys on a calculator to determine *approximate values* of these irrational numbers.

?

?

There are other irrational numbers besides radicals; for example, π.

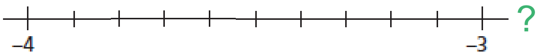
We can approximate the location of an irrational number on a number line. If we do not have a calculator, we use perfect powers to estimate the value. For example, to locate $\sqrt[3]{-50}$ on a number line, we know that $\sqrt[3]{-27} = -3$ and $\sqrt[3]{-64} = -4$.

Guess: $\sqrt[3]{-50} \doteq -3.6$ Test: $(-3.6)^3 = -46.656$

Guess: $\sqrt[3]{-50} \doteq -3.7$ Test: $(-3.7)^3 = -50.653$

This is close enough to represent on a number line.

Since $(-3.7)^3 = -50.653$, then $\sqrt[3]{-50}$ is slightly greater than -3.7, so mark a point to the right of -3.7 on the number line.



Example 1 Classifying Numbers

Tell whether each number is rational or irrational. Explain how you know.

a) $-\frac{3}{5}$ b) $\sqrt{14}$ c) $\sqrt[3]{\frac{8}{27}}$

SOLUTION

CHECK YOUR UNDERSTANDING

Example 2 Ordering Irrational Numbers on a Number Line

Use a number line to order these numbers from least to greatest.

$\sqrt[3]{13}$, $\sqrt{18}$, $\sqrt{9}$, $\sqrt[4]{27}$, $\sqrt[3]{-5}$

SOLUTION

CHECK YOUR UNDERSTANDING

4.2 Irrational Numbers

Number line Activity

With a partner, or by yourself, place the numbers from LEAST to GREATEST.

Let's see who can get done first!

(When you are done, call me over so that I can check it)

33	$\sqrt{9}$	1.45	-1.23	-4.5
$\sqrt[3]{13}$	0	$\sqrt{36}$	$\sqrt[3]{-5}$	π
2/3	-8/4	-3	$(5)^3$	8/3
$(-2)^3$	$\sqrt{144}$	1002	-5.68	$\sqrt{64}$
$\sqrt{18}$	9 1/2	-1.22	6/7	$2 \times 4 \div \sqrt{4}$

$(-2)^3$ | -5.68 | -4.5 | -3 | $-8/4$ | $\sqrt[3]{-5}$ | -1.23 | -1.22 | 0 | $2/3$ | $6/7$ | 1.45

$\sqrt[3]{13}$ | $8/3$ | $\sqrt{9}$ | π | $2 \times 4 \div \sqrt{4}$ | $\sqrt{18}$ | $\sqrt{36}$ | $\sqrt{64}$ | $9\ 1/2$ | $\sqrt{144}$ | 33 | $(5)^3$ | 1002

1. Choose only the rational numbers
2. Choose only the whole numbers
3. Choose only the integers
4. Choose only the irrational numbers
5. Choose only the natural numbers
6. Choose only the real numbers